

Hong Kong Mathematics Olympiad (2015/16)

Heat Event (Individual)

香港数学竞赛 (2015/16)

初赛项目(个人)

除非特别声明，答案须用数字表达，并化至最简。

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

甲部

Part A

1. 计算 $0.125^{2016} \times (2^{2017})^3$ 的值。

Find the value of $0.125^{2016} \times (2^{2017})^3$.

2. 已知方程 $\begin{cases} x_1 + x_2 = x_2 + x_3 = x_3 + x_4 = \cdots = x_{2014} + x_{2015} = x_{2015} + x_{2016} = 1 \\ x_1 + x_2 + x_3 + \cdots + x_{2015} + x_{2016} = x_{2016} \end{cases}$, 求 x_1 的值。

Given the equations $\begin{cases} x_1 + x_2 = x_2 + x_3 = x_3 + x_4 = \cdots = x_{2014} + x_{2015} = x_{2015} + x_{2016} = 1 \\ x_1 + x_2 + x_3 + \cdots + x_{2015} + x_{2016} = x_{2016} \end{cases}$, find the value of x_1 .

3. 有多少个数 x 使得 $\sqrt{2016 - \sqrt{x}}$ 为整数?

How many x are there so that $\sqrt{2016 - \sqrt{x}}$ is an integer?

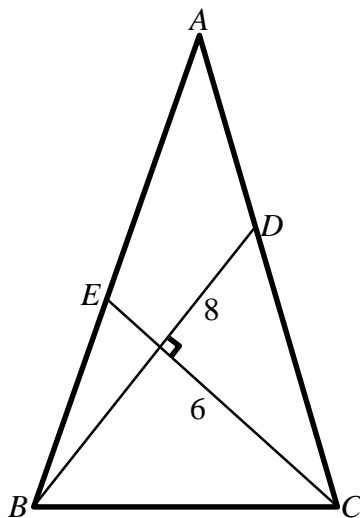
4. 若 x 、 y 为整数，有多少对 x 、 y 且满足 $(x+1)^2 + (y-2)^2 = 50$ 。

If x , y are integers, how many pairs of x , y are there which satisfy the equation $(x+1)^2 + (y-2)^2 = 50$.

5. 63 个连续整数的和是 2016，求紧接该 63 个连续整数后的 63 个连续整数的和。
The sum of 63 consecutive integer is 2016 , find the sum of the next 63 consecutive integers.
6. 已知 8 个整数的平均数、中位数、分布域及唯一众数均为 8。若 A 为该 8 个整数中的最大数，求 A 的最大值。
Given that the mean, median, range and the only mode of 8 integers are also 8 . If A is the largest integer among those 8 integers, find the maximum value of A .
7. 在整数 1 至 500 之间出现了多少个数字「2」？
How many '2's are there in the numbers between 1 to 500 .
8. 某数的 16 进制位数是 1140。而同一数字的 a 进制位数是 240，求 a 。
A number in base 16 is 1140 . The same number in base a is 240 , what is a ?
9. P 点的极坐标为 $(6, 240^\circ)$ 。若 P 向右平移 16 单位，求 P 的像与极点之间的距离。
The polar coordinates of P are $(6, 240^\circ)$. If P is translated to the right by 16 units, find the distance between its image and the pole.

10. 如图一，在 $\triangle ABC$ 中， BD 和 CE 分别是 AC 和 AB 两边上的中线，且 $BD \perp CE$ 。已知 $BD = 8$ ， $CE = 6$ ，求 $\triangle ABC$ 的面积。

As shown in Figure 1, BD and CE are the medians of the sides AC and AB of $\triangle ABC$ respectively, and $BD \perp CE$. Given that $BD = 8$, $CE = 6$, find the area of $\triangle ABC$.



图一
Figure 1

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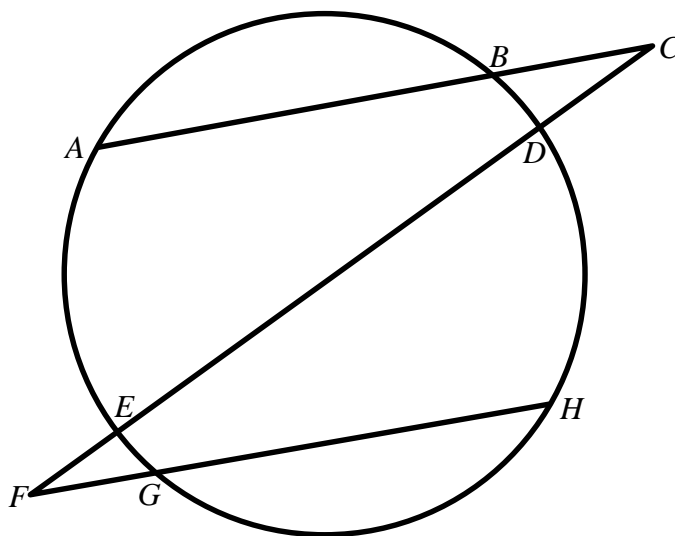
Part B

11. 已知方程 $100[\log(63x)][\log(32x)] + 1 = 0$ 有两个相异的实数根 α 及 β ，求 $\alpha\beta$ 的值。

It is known that the equation $100[\log(63x)][\log(32x)] + 1 = 0$ has two distinct real roots α and β . Find the value of $\alpha\beta$.

12. 如图二所示, ABC , $CDEF$ 及 FGH 皆为直线, 且 $ABC \parallel FGH$ 。 $AB = 42$, $GH = 40$, $EF = 6$ 及 $FG = 8$ 。已知 ABC 与 FGH 之间的最短距离为 41, 求 BC 。

As shown in Figure 2, ABC , $CDEF$ and FGH are straight lines, $ABC \parallel FGH$. $AB = 42$, $GH = 40$, $EF = 6$ and $FG = 8$. Given that the distance between ABC and FGH is 41, find BC .



图二

Figure 2

13. 设 A 、 B 和 C 为三个数字。利用这三个数字组成的三位数有以下性质:

- (a) ACB 可以被 3 整除;
- (b) BAC 可以被 4 整除;
- (c) BCA 可以被 5 整除; 及
- (d) CBA 的因子数目为单数。

求三位数 ABC 。

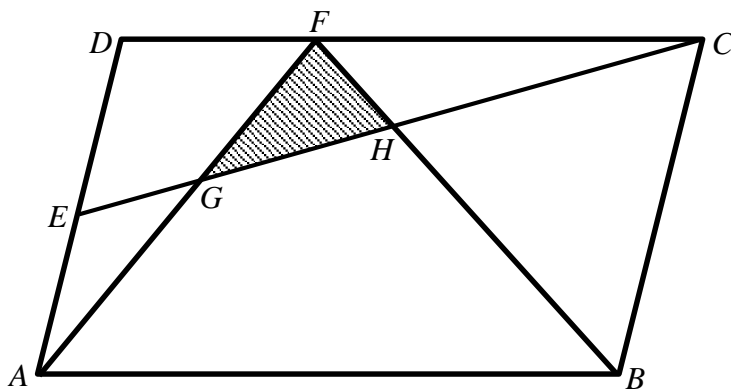
Let A , B and C be three digits. The number formed by these three digits has the following properties:

- (a) ACB is divisible by 3 ;
- (b) BAC is divisible by 4 ;
- (c) BCA is divisible by 5 ; and
- (d) CBA has an odd number of factors.

Find the 3-digit number ABC .

14. 如图三，在图中， $ABCD$ 为一平行四边形。 E 为 AD 的中点及 F 是 DC 上的点且满足 $DF:FC=1:2$ 。 FA 及 FB 分别相交 EC 于 G 及 H 。求 $\frac{ABCD \text{ 的面积}}{\triangle FGH \text{ 的面积}}$ 的值。

As shown in Figure 3, $ABCD$ is a parallelogram. E is the mid-point of AD and F is a point on DC such that $DF:FC=1:2$. FA and FB intersect EC at G and H respectively. Find the value of $\frac{\text{Area of } ABCD}{\text{Area of } \triangle FGH}$.



图三

Figure 3

15. 已知数列 $\{a_n\}$ 其中 $a_{n+2} = a_{n+1} - a_n$ 。若 $a_2 = -1$ 及 $a_3 = 1$ ，求 a_{2016} 的值。
Given a sequence $\{a_n\}$, where $a_{n+2} = a_{n+1} - a_n$. If $a_2 = -1$ and $a_3 = 1$, find the value of a_{2016} .

完
END